Plasma Physics

Collisions

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Structure

Collisions

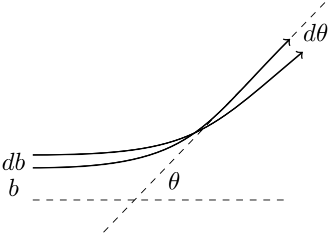
Rutherford scattering

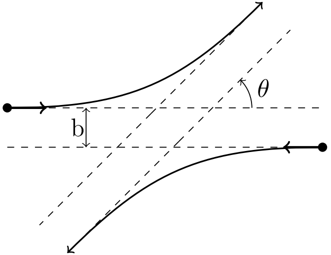
Effective cross-section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{2\pi b|\mathrm{d}b\ |}{2\pi|\sin(\theta)\,\mathrm{d}\theta\ |} = \frac{b}{\sin(\theta)}\left|\frac{\mathrm{d}b}{\mathrm{d}\theta}\right|$$

Rutherford scattering:

$$\begin{aligned} b &= b_c \cot(\theta/2) \\ b_c &= \frac{Ze^2}{4\pi\varepsilon_0 m v_0^2} \\ \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= \frac{b_c^2}{4\sin(\theta/2)^4} \end{aligned}$$





Collisions

Small and large angle collisions

Impact parameter for $\theta = \pi/2$:

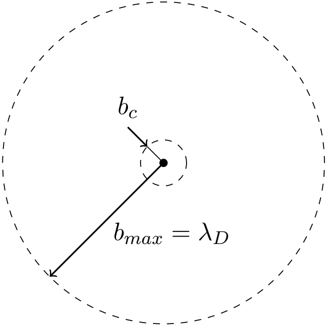
$$b_c = \frac{Ze^2}{4\pi\varepsilon_0 m v_0^2}$$

Maximumu impact parameter:

$$b_{max} = \lambda_D$$

For thermal velocity $\langle m v_0^2 \rangle = 3k_B T$:

$$\frac{b_c}{b_{max}} = \frac{Z}{12\pi}\frac{1}{n\lambda_D^3} \propto \frac{1}{N_D^3}$$



Collisions

Collision frequency

Collision frequency determines the thermalization time.

Single collision at small θ:

- small energy exchange
- wide cross section

λ_c: *Mean free path* before a *total* deviation of $\pi/2$.

Single collision at large θ:

- large energy exchange
- small cross section

$\lambda_{\pi/2}$: *Mean free path* before a deviation of $\pi/2$ in a single collision.

In order to determine the thermalization time we need to:

- find the most effective energy exchange mechanism;
- determine the time scale for single specie thermalization;
- determine the time cross-specie thermalization.

Collisions

Small angle mean free path

$$\langle \Delta v_x^2 \rangle = N \langle (\Delta v_x)_i^2 \rangle$$

$$(\Delta v_x)_i^2 \simeq v^2 \sin^2(\theta)$$

$$\langle (\Delta v_x)_i^2 \rangle = \frac{\int_{b_c}^{b_{max}} 2\pi b\, (\Delta v_x)_i \,\mathrm{d}b}{\int_{b_c}^{b_{max}} 2\pi b \mathrm{d}b} \simeq \frac{8\pi b_c^2 v^2 \Lambda}{\int_{b_c}^{b_{max}} 2\pi b \mathrm{d}b}$$

$$\boxed{\textit{Coulombian Logarithm: } \Lambda = \ln(\lambda_D / b_c)}$$

$$N = \left(n \int_{b_c}^{b_{max}} 2\pi b \mathrm{d}b \right) \lambda_c$$

$$\boxed{\textit{Small deflection mean free path: } \lambda_c = \frac{1}{8\pi n b_c^2 \Lambda}}$$

where a for a deflection of $\pi/2$ it has been considered $\Delta v_x = v$.

Collisions

$\pi/2$ mean free path

$$\begin{aligned} N_{\pi/2} = 1 &= n\lambda_{\pi/2} \int_0^{b_c} 2\pi b \mathrm{d}b \\ &= n\lambda_{\pi/2}\pi b_c^2 \end{aligned}$$

$$\boxed{\textit{\pi/2 deflection mean free path: :}\ \lambda_{\pi/2} = 8\Lambda\lambda_c}$$

In conclusion, $\lambda_{\pi/2} \ll \lambda_c$ which indicates that **the largest part of energy exchange happens due to small θ deflections**.

Collisions

Collision frequency and thermalization

- Effects of "large angle deflection" can be neglected (this is inherent to the Coulomb range).
- Temperature dependence for $\lambda_c \sim 1/nb_0^2$ is included in the b_0 factor.
- For thermal particles it holds:

$$b_0 \sim \left(\frac{Z}{4\pi\varepsilon_0}\right)\frac{e^2}{\mu v_t^2}$$

The three possible collisions are considered (restrictions apply):

electron-ion

$$\mu \simeq m_e$$
$$v \simeq v_e = v_{te}$$
$$v_{te} = (3k_B T_e/m_e)^{1/2}$$

ion-ion

$$\mu = m_i/2$$
$$v = v_{ti}$$
$$v_{ti} = (3k_B T_i/m_i)^{1/2}$$

electron-electron

$$\mu = m_e/2$$
$$v = v_{te}$$

(Not different from electron-ion)

Collisions

Collision frequency and thermalization

Electron-Ion

$$\tau_{ei} = \frac{\lambda_c}{v_{te}} = \frac{1}{\nu_{ei}} = \left[\frac{n_i Z^2 e^4 \Lambda}{4\pi\varepsilon_0^2 m_e^{1/2} (k_B T_e)^{3/2}}\right]^{-1}$$

Ion-Ion

$$\tau_{ii} = \frac{\lambda_c}{v_{ti}} = \frac{1}{\nu_{ei}} = \left[\frac{n_i Z^4 e^4 \Lambda}{4\pi\varepsilon_0^2 m_i^{1/2} (k_B T_i)^{3/2}}\right]^{-1}$$

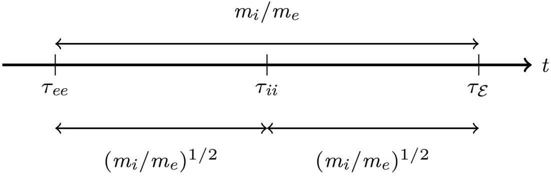
Electron-Electron

$$\tau_{ee} = \frac{\lambda_c}{v_{te}} = \frac{1}{\nu_{ee}} = \left[\frac{n_e e^4 \Lambda}{4\pi\varepsilon_0^2 m_e^{1/2} (k_B T_e)^{3/2}}\right]^{-1}$$

$$\frac{\tau_{ei}}{\tau_{ii}} \simeq \left(\frac{T_e}{T_i}\right)^{3/2} \left(\frac{m_e}{m_i}\right)^{1/2}$$

(Note: according to previous definition, $\tau_{ei} \neq \tau_{ie}$!)

On Thermalization time



- Due to e-e collisions ($\tau \approx \tau_{ee}$), electron population reaches maxwellian distribution in short timescale;
- same applies for ions, but on a longer time scale, due to slower collision rate ($\tau \approx \tau_{ii}$);
- Crossed specie collision happens on fast time ($\tau_{ee} \approx \tau_{ei}$). However (m_i/m_e) collisions are needed to exchange of an amount of energy in the order of the average:

$$\boxed{\frac{\Delta\mathcal{E}}{\mathcal{E}} \approx \frac{m_e}{m_i}}$$